

The DC Grotto Compass Course

by Robert Thrun

The Study

Several members of the D.C. Grotto had been speculating about the accuracy of the instruments that we use for our cave surveys. In March 1981, we set up a compass course to determine the accuracy and repeatability of our compass readings. The course consisted of a central wooden post and ten posts at different angles around it. The angles between posts were established by trilateration. Nine people used 31 compasses, including some compasses that would not normally be used for cave surveying. A full set of readings consisted of ten readings from the center post to each of the perimeter posts and ten readings from the perimeter posts to the center. The tests were made on three consecutive weekends, March 15, March 21 and 22, and March 28 and 29.

The compass course would best be termed a study. We were not running an experiment to prove a hypothesis or determine any particular numbers. We just collected data with the intention of analyzing it later to see if any trends were apparent. There were two talks given at NSS conventions in consecutive years, but nothing has been published until this article.

A least-squares fit to each ten readings gave magnetic north relative to the posts, the consistency of readings, and any eccentricity error. The following were obtained:

- The standard deviation (S.D.) for one shot by various persons with different compass types.
- The average magnetic north for the entire study.
- The standard deviation for the difference between magnetic north for each compass and average magnetic north.
- The standard deviation for sets of 10 readings by person and type of compass.
- The effect of tripod mounting for a pocket transit (Brunton compass).

The Site

Tom Kaye found a park, the Washington Sailing Marina, that was a suitable site for some tests. It is along the Potomac River in Arlington, Virginia, south of Washington D.C. It has parking areas for boat trailers and vehicles, large areas of lawn, and drive-

ways through the park. The thing that made it suitable for compass tests was the fact that the driveways were bordered by short, closely spaced, wooden posts that had no metal in them. We set up our test course in an area as far from the vehicles and trailers as possible. One post was chosen as the center post. Ten other posts were chosen at approximately equal angular intervals around the center post. The layout of the course is given in Table 1 and Figure 1. The angles from the center to the perimeter posts are relative to an average magnetic north, which will be explained later.

Table 1. Distances and magnetic angles to each of the posts.

| To post | Distance (meters) | Angle (degrees) |
|---------|-------------------|-----------------|
| 1 | 10.388 | 15.37 |
| 2 | 12.984 | 49.54 |
| 3 | 18.018 | 81.80 |
| 4 | 10.936 | 105.54 |
| 5 | 13.981 | 148.67 |
| 6 | 8.696 | 187.17 |
| 7 | 11.400 | 209.15 |
| 8 | 21.196 | 255.00 |
| 9 | 18.837 | 286.85 |
| 10 | 13.951 | 317.10 |

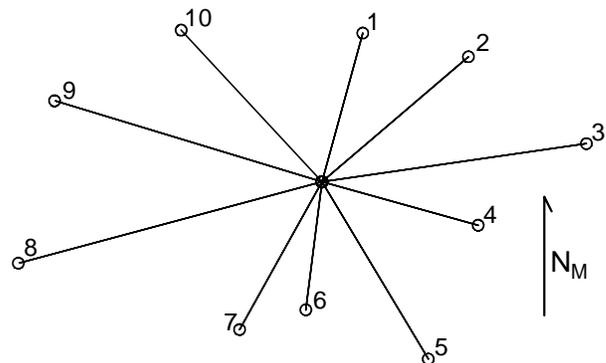


Figure 1. Layout of the posts that were used.

Running the Course

In running through the compass course, a person would take a compass shot from the center post to each of the perimeter posts. Then he or she would go to the perimeter and take ten shots to the center. More than one person could be on the perimeter at different posts at the same time. We even found that two people could use the center post if one of them backed away from it and sighted over it. We often had three people taking readings at the same time. The readings would be called out to a notetaker, usually Linda Baker (now Devine), who would get everything written down in the right place. Occasionally, the noise of an airplane taking off from nearby National Airport would suspend operations. At first the readings were written in cave surveying notebooks, but later Linda made special forms for recording the data. More than 1600 compass readings were recorded.

For accuracy, the sightings were made between quarter-inch dowels that were placed in the top of the posts. The sighting conditions were nearly ideal. The posts were short enough so that a Brunton tripod could be placed over them. The posts were at a convenient height for a reading while kneeling. The course was nearly level. The steepest shot was 1.5 degrees from horizontal.

Participants and Instruments

Nine people made readings using 31 compasses. There were 13 metal-bodied Suuntos, 5 Brunton-style compasses, 5 plastic-bodied Suuntos, and 7 miscellaneous compasses. The persons and instruments were code-numbered for the purpose of this report. The persons were P1 to P9. The metal Suuntos were S1 to S13, the Bruntons were B1 to B6, The plastic Suuntos were PL1 to PL5, and the miscellaneous compasses were M1 to M7.

The metal-bodied Suuntos, S1 to S13, were the KB-14 model that most cave surveyors are familiar with. S13 was graduated in grads (400 to a circle) rather than degrees.

B1 and B2 were the same compass, given different numbers for tripod-mounted and handheld use. B1/B2 was a Keuffel & Esser Pocket Transit. This brand was the best constructed of the Brunton-style instruments at the time the course was run. It had a simple long magnetic needle. B3, B4 and B5 were recent manufacture Ainsworth Bruntons. (Brunton was a trademark of Ainsworth, and is now a trademark of the Brunton Company.) The Ainsworth

Bruntons had a brass needle with two magnets on outriggers alongside the needle. B3 was used only tripod-mounted. B6 was an old military Brunton marked in mils (6400 to a circle). It was in poor condition. The needle stuck very noticeably, even with tapping.



Figure 2. Metal-bodied (left) and plastic Suuntos



Figure 3. Tripod-mounted Keuffel and Esser Pocket Transit

The plastic-bodied Suuntos, PL1 to PL5, were the KB-20 model, a lower-cost version of the metal-bodied Suuntos. They were similar in shape and size to the metal Suuntos. This model is no longer made, having been replaced by a smaller model. Their dials were marked in whole degrees, while the metal Suuntos were marked in half degrees.

M1 was a French-made instrument, called a "Hockey Puck", that had been used for cave surveying by its owner. It had a clear plastic body, a clear plastic dial with two short bar magnets clipped to the underside, a prism and magnifier mounted on top, and what looked like a small rubber tire wrapped around the outside. It had no hairline. Instead the magnifier and

prism were placed on the far side of the dial with the magnification cleverly chosen so that the dial graduations appeared stationary as the instrument rotated. M2 was an "Omega Hand Bearing Compass" that was identical to the Hockey Puck except for the brand name. These instruments were difficult to read on the slight downslopes that were encountered on the course. Most people read these compasses one-eyed.



Figure 4. The Omega Hand Bearing Compass.

M3 was a Swiss-made Meridian geologist's compass of very good construction. It is what is generally called a prismatic compass. It used a prism to read the near side of the dial and had a hairline in a folding sight.. Its dial was a rotating aluminum disk with two short bar magnets clipped to the underside.



Figure 5. The Meridian geologist's compass.

M4 was a Ritchie marine compass. The user held it up to his eye, looked through a viewfinder, and pressed a button to light up a red sighting dot. (It requires nonmagnetic AAA size batteries.) At the same time he pressed another button to allow the dial, which was not visible, to rotate freely. After the dial had time to settle down, he released the button to lock the dial and then took the compass down from his eye to look at the dial from the top and make a reading. The clamping action sometimes slightly disturbed the dial.

M5 was a Silva Model 50. Its body had two plastic clamshell halves that pulled apart to reveal the compass dial. The dial was read with a prism and magnifier on the far side as on the Hockey Puck, and it also had an out-of focus hairline. This model is no longer made.



Figure 6. The Silva Model 50 compass.

M6 was a Weems and Plath marine hand bearing compass. It was large, had a wooden handle under it, and came in a fitted wooden case.

M7 was a Wilkie lensatic compass, sold as a hiking compass. It had a fixed lens in the side of the body and a hairline in the lid. It was of better quality than the copies of US Army lensatic compasses that are readily available.

The Sisteco compass and clinometer were not on the market at the time of the study. These instruments, which greatly resemble the metal-bodied Suuntos, are now made by Silva and sold in the US with a Brunton label.

Simple Analysis

The simplest analysis is to find the mean and standard deviation for many readings between pairs of posts. This represents the variation that we would get if we had random individuals using random

instruments. Many survey projects operate in this way. Different people do the survey, each team with their own instruments, and their readings are taken at face value. Tables 2 to 5 give this analysis. Not all the readings are represented in Tables 2 to 5. If a set of 10 readings was judged bad for any reason, even for one blunder or bad shot, then all the shots for that set were omitted.

The identification of bad data sets was done by sinusoidal curve fits as will be described later. Evidently not all bad sets were identified. The high S.D. for shots 7–center with metal Suuntos was probably due to a boat trailer near Post 7 on one day.

The angles presented here are not the same as the angles in Table 1. In Table 1, each compass was counted only once. In Tables 2 to 5, each compass is counted by the number of times it was used. Tables 2 to 5 give the standard deviations that we get from different persons repeating the same shot with different uncalibrated compasses. These standard deviations include effects from the instrument and person differences.

The Bruntons seemed to be better than the Suuntos in this analysis. The least-squares analysis showed that the Bruntons were more alike in their magnetic north than the Suuntos. This may be due to the instruments' design, it may be because the Brunton readers were better, or it may be due to the fact that there were only 4 good Bruntons in the sample. But there were only 5 plastic Suuntos and they had about as much variation as the metal Suuntos. The standard deviations from station to station were not as consistent as I expected.

Table 2. Analysis by Station for Metal Suuntos

| 33 shots from center | | | 28 shots to center | | |
|----------------------|-----------------|-----------|--------------------|-----------------|-----------|
| To post | Average azimuth | Std. dev. | From post | Average Azimuth | Std. dev. |
| 1 | 15.36 | 1.40 | 1 | 194.99 | .88 |
| 2 | 49.11 | .85 | 2 | 229.34 | 1.24 |
| 3 | 81.48 | .81 | 3 | 261.57 | .69 |
| 4 | 105.33 | .81 | 4 | 284.81 | .99 |
| 5 | 148.51 | 1.15 | 5 | 328.46 | 1.44 |
| 6 | 186.98 | .87 | 6 | 7.17 | 1.05 |
| 7 | 209.01 | .81 | 7 | 29.62 | 1.52 |
| 8 | 255.07 | .85 | 8 | 74.80 | 1.21 |
| 9 | 286.96 | 1.31 | 9 | 106.42 | .88 |
| 10 | 316.43 | .92 | 10 | 137.03 | 1.17 |

Table 3. Analysis by Station for Bruntons

| 12 shots from center | | | 11 shots to center | | |
|----------------------|-----------------|-----------|--------------------|-----------------|-----------|
| To post | Average azimuth | Std. dev. | From post | Average azimuth | Std. dev. |
| 1 | 15.09 | .33 | 1 | 195.45 | .27 |
| 2 | 49.99 | .47 | 2 | 229.21 | .85 |
| 3 | 81.66 | 1.05 | 3 | 261.97 | .52 |
| 4 | 105.38 | .48 | 4 | 285.38 | .76 |
| 5 | 148.31 | .57 | 5 | 328.96 | .60 |
| 6 | 186.89 | .61 | 6 | 7.54 | .55 |
| 7 | 209.35 | .55 | 7 | 29.84 | .32 |
| 8 | 254.37 | .43 | 8 | 74.48 | .67 |
| 9 | 286.38 | .83 | 9 | 106.58 | .33 |
| 10 | 316.56 | .61 | 10 | 137.65 | .39 |

Table 4. Analysis by Station for Plastic Suuntos

| 19 shots from center | | | 18 shots to center | | |
|----------------------|-----------------|-----------|--------------------|-----------------|-----------|
| To post | Average azimuth | Std. dev. | From post | Average azimuth | Std. dev. |
| 1 | 15.99 | 1.47 | 1 | 195.77 | .38 |
| 2 | 49.67 | .87 | 2 | 229.81 | 1.13 |
| 3 | 81.94 | .94 | 3 | 261.95 | .73 |
| 4 | 105.85 | .82 | 4 | 285.36 | .90 |
| 5 | 148.86 | .83 | 5 | 329.39 | .74 |
| 6 | 187.57 | .80 | 6 | 7.63 | 1.20 |
| 7 | 209.33 | 1.05 | 7 | 30.32 | 1.20 |
| 8 | 255.55 | .92 | 8 | 75.73 | .90 |
| 9 | 287.43 | .95 | 9 | 107.13 | 1.00 |
| 10 | 316.61 | .90 | 10 | 137.83 | .97 |

Table 5. Analysis by Station for Miscellaneous Compasses

| 7 shots from center | | | 6 shots to center | | |
|---------------------|-----------------|-----------|-------------------|-----------------|-----------|
| To post | Average Azimuth | Std. dev. | From post | Average azimuth | Std. dev. |
| 1 | 15.81 | .58 | 1 | 195.85 | .23 |
| 2 | 50.13 | .86 | 2 | 230.21 | .87 |
| 3 | 82.27 | .91 | 3 | 262.92 | .45 |
| 4 | 105.79 | .72 | 4 | 286.53 | .84 |
| 5 | 149.40 | 1.29 | 5 | 329.55 | 1.35 |
| 6 | 188.10 | .94 | 6 | 7.97 | 1.09 |
| 7 | 210.23 | .84 | 7 | 31.13 | 1.15 |
| 8 | 256.00 | .80 | 8 | 76.15 | .60 |
| 9 | 287.63 | 1.12 | 9 | 107.13 | .56 |
| 10 | 317.57 | 1.13 | 10 | 138.05 | .50 |

Sources of Error

Bias error is a consistent difference between the direction that the compass indicates and the actual direction with respect to the magnetic field. On a Brunton with a long needle, this is caused by a bent needle or by the sights not being aligned with the markings on the compass dial. On compasses with a rotating dial, bias error can be caused by the magnet or magnets not being aligned with the dial. On compasses with two magnets, the magnets must be balanced in location and strength.

Eccentricity error is a sinusoidally varying error as the compass is rotated through a full circle. It is caused by an off-center pivot. The eccentricity error should average zero over 360 degrees.

A least-squares curve fit was used to get the eccentricity error for each set of ten readings. The eccentricity error can be found by a fit to an equation of the form:

$$\varphi = \theta + c + a \cos(\theta - b)$$

where

φ = compass reading

θ = angle relative to average magnetic north

This is the independent variable.

a = amount, in degrees, of eccentricity error

b = direction of eccentricity error

c = compass bias error

The use of the cosine and the minus sign on b means, for instance, that if b is 45 degrees, then the maximum positive error occurs at 45 degrees. The error in the location of the pivot point is 90 degrees ahead of b . If the maximum error is at 45 degrees, then the pivot is displaced toward the 135 degree direction. This equation is nonlinear in the parameter b , making the usual linear least-squares fit method inapplicable. Instead, a linear fit to the equation

$$\varphi = \theta + c + A \cos \theta + B \sin \theta$$

was made. This is linear in the parameters A , B , and c . This was transformed by trigonometric identities to the desired form. The results of the curve fits for 162 sets of ten readings are given in an appendix.

The least-squares curve fitting process can fit a sinusoidal curve to any set of data. Even if there is no eccentricity error in a compass, a sinusoidal curve can be fitted to the random errors. An actual eccentricity error may be identified by several clues. The indicated eccentricity should be large and always in the same general direction for a particular compass,

and the standard deviations for a sinusoidal fit should be significantly less than for a linear fit and less than the eccentricity.

Some of the compass readers made reading blunders, but they were usually caught immediately by the notetaker, who had all of the previous readings to compare with. No record was kept of the blunders. Some blunders slipped through anyway. Another problem was caused by a boat trailer being parked close to one of the posts we used on one day.

In analyzing the data, it is apparent that some persons read the compasses more accurately than others. Since different people read different compasses, we are unable to determine if a bias error was due to the compass or to the reader. Effectively, we assumed that the personal bias errors were zero because we can't measure them. Other errors, including external influences, fluctuations in the Earth's magnetic field, errors in marking the compass dial, and any other errors that may be present are lumped together as random error.

Angles Between Posts

The distances between aim points were taped in order to establish the relative angles by trilateration. A fiberglass tape of the type common in cave surveying was used to make measurements during the period of the compass readings. The ten trilaterated angles about the center post totaled 360.0088 degrees. This would amount to a 0.2 to 0.4 millimeter measurement error between each pair of perimeter posts. The angles between posts were adjusted by an appropriate amount. These angles were then considered fixed for the least squares fits and the determination of average magnetic north.

Grand Average North

A measurement between one of the perimeter posts and the center was sufficient to determine the magnetic angles for all the other posts just by adding the angles between them to the angle for the one reading. A set of ten compass readings was, in effect, ten independent measurements of magnetic north.

The angle to the first post was chosen as an arbitrary origin. For each of the ten shots in a set, we got the difference between the trilaterated angle and the compass reading. We averaged these to get the average magnetic north relative to the set of posts for each set of ten readings.

Since each compass has its own bias error, all the sets of readings for each compass were averaged to get the average magnetic north for each compass. This did not include any eccentricity correction. The average north values for each compass were averaged to get the Grand Average North for the entire study. Since the metal Suuntos and Bruntons seemed better than the other compasses, only they were used to get Grand Average North. B2 was omitted because it was represented more accurately as B1. Using the north from the sinusoidal fits for the same compasses would move magnetic north 0.007 degree to the west.

Any set of ten readings that had an identifiable blunder, a reading that was disturbed by a vehicle, or simply had a high standard deviation from the curve fit was not used in getting average magnetic north or for other analyses.

Bias errors

Table 6 gives the average bias error for each of the compasses. The bias error was considered to be the average difference between Grand Average North and average magnetic north for one compass for a linear fit. The standard deviations shown in Table 6 are for sets of ten readings, not for individual readings. It indicates how well different sets of ten readings with a compass agreed with each other. A large standard deviation does not necessarily mean that a particular compass is less consistent. There is a slight tendency for the compasses that were read the least number of times to have the smallest standard deviations between sets of readings, indicating that there are differences between readers.

It would be much better to compare the compasses to the actual magnetic north, rather than to the average of the compasses, but we had no absolute way of establishing magnetic north. We could have gotten true north by sighting to the Washington Monument, more than 4 miles to the north.

The average magnetic north for groups of compasses and the standard deviations between the compasses in each group are given in Table 7. The average magnetic north for S1-S13, B1, and B3-B5 was taken to be the average magnetic north for the entire study and was arbitrarily taken to be zero. The standard deviations in Table 7 are for the differences between compasses, not for sets of readings or for individual readings.

Table 6. The difference (bias) between Grand Average North of all Bruntons and metal Suuntos and the average for each compass. N is the number of sets of ten readings.

| Compass | N | Avg. bias | Std. dev. |
|---------|----|-----------|-----------|
| S1 | 3 | +0.27 | 0.16 |
| S2 | 3 | +0.19 | 0.38 |
| S3 | 3 | +0.54 | 0.09 |
| S4 | 7 | +0.28 | 0.38 |
| S5 | 11 | -1.19 | 0.82 |
| S6 | 4 | +0.70 | 0.35 |
| S7 | 3 | -1.17 | 0.99 |
| S8 | 3 | +0.65 | 0.25 |
| S9 | 8 | -0.62 | 0.73 |
| S10 | 7 | -0.15 | 0.45 |
| S11 | 3 | +0.24 | 0.44 |
| S12 | 3 | +0.18 | 0.31 |
| S13 | 2 | +0.17 | 0.09 |
| B1 | 6 | +0.07 | 0.14 |
| B2 | 8 | -0.12 | 0.29 |
| B3 | 2 | -0.01 | 0.12 |
| B4 | 4 | -0.04 | 0.44 |
| B5 | 3 | -0.10 | 0.63 |
| B6 | .. | bad | ... |
| PL1 | 4 | +0.32 | 0.38 |
| PL2 | 8 | +0.32 | 0.09 |
| PL3 | 8 | +0.11 | 0.19 |
| PL4 | 4 | +0.82 | 0.30 |
| PL5 | 12 | +0.48 | 0.68 |
| M1 | 4 | +0.55 | 0.30 |
| M2 | 4 | +1.54 | 0.44 |
| M3 | 2 | +0.19 | 0.07 |
| M4 | 1 | +0.28 | ... |
| M5 | 2 | +0.66 | 0.04 |
| M6 | .. | bad | ... |
| M7 | .. | bad | ... |

Table 7. The differences between Grand Average North for the entire study and various groups of compasses.

| Group of compasses | Avg. North | Std. dev. |
|-----------------------|--------------------|-----------|
| S1-S13, B1, and B3-B5 | Zero by definition | 0.54 |
| S1-S13 | +0.007 | 0.63 |
| B1, B3-B5 | -0.020 | 0.071 |
| PL1-PL5 | +0.41 | 0.26 |
| M1-M5 | +0.67 | 0.54 |

The Curve Fits

The program to process the data did both a linear fit and a sinusoidal fit to the differences between the compass readings and the angles given by trilateration and Grand Average North. The angles were shown in Table 1. The linear fit is simply an average of the differences. The standard deviation from the curve fits represents the random errors from a set of ten readings. These errors are what we would have if the same person made all the readings with the same compass. The average values of the standard deviations are shown in Table 8.

Compass readers P4 and P5 normally read to 0.1 degree. This contributed to accuracy under these

ideal conditions. Most others read to 0.5 degree resolution with some readings to 0.25 degrees. The metal Suuntos were marked in 0.5 degree increments and the plastic Suuntos were marked in 1 degree increments. Both were read to 0.5 degree precision by most readers, except for P4 and P5.

The metal Suuntos did slightly better than the Bruntons as read by the same person. This may be due to the magnifier allowing more precision in reading. It should be remembered that this course was level. The Bruntons might be better on a test where there are steeper shots. The plastic Suuntos were poorer than either the metal Suuntos or Bruntons.

Table 8 . Average standard deviations grouped by persons. The standard deviations are about the compass bias for the set of ten readings, indicating the accuracy of the compass/person combination if compass bias could be accurately determined.

| Person | All Compasses | | | Metal Suuntos | | | Bruntons | | | Plastic Suuntos | | | Miscellaneous Compasses | | |
|--------|---------------|----------|----------|---------------|----------|----------|----------|----------|----------|-----------------|----------|----------|-------------------------|----------|----------|
| | N | LIN avg. | SIN avg. | N | LIN avg. | SIN avg. | N | LIN avg. | SIN avg. | N | LIN avg. | SIN Avg. | N | LIN avg. | SIN avg. |
| P1 | 6 | .88 | .64 | 2 | .83 | .62 | ... | ... | ... | 2 | 1.03 | .76 | 2 | .78 | .55 |
| P2 | 18 | .96 | .82 | 10 | .95 | .86 | 2 | .84 | .75 | 4 | 1.10 | .82 | 2 | .89 | .74 |
| P3 | 30 | .73 | .59 | 13 | .56 | .45 | 4 | .65 | .55 | 13 | .93 | .73 | ... | ... | ... |
| P4 | 32 | .45 | .36 | 12 | .29 | .24 | 13 | .50 | .38 | ... | ... | ... | 7 | .61 | .54 |
| P5 | 9 | .66 | .45 | 1 | .48 | .38 | 4 | .76 | .49 | 4 | .61 | .44 | ... | ... | ... |
| P6 | 10 | .77 | .42 | 2 | .49 | .44 | ... | ... | ... | 8 | .84 | .42 | ... | ... | ... |
| P7 | 13 | .78 | .60 | 9 | .79 | .62 | ... | ... | ... | 4 | .76 | .55 | ... | ... | ... |
| P8 | 8 | .91 | .68 | 6 | .84 | .70 | ... | ... | ... | 2 | 1.12 | .63 | ... | ... | ... |
| P9 | 8 | .66 | .57 | 5 | .68 | .58 | ... | ... | ... | 1 | .47 | .46 | 2 | .68 | .61 |

Additionally, there are enough data to determine the effect of tripod mounting on the accuracy of a Brunton. A magnifier was used to read the tripod-mounted Brunton to 0.1 degree, which partly accounts for the better performance of the tripod-mounted instrument. Table 9 shows the average values of the standard deviations. Each standard deviation is from a set of ten readings with either a linear or sinusoidal fit.

Table 9. Effect of handholding and tripod mounting on the average standard deviation with a Brunton.

| Person | N | LIN avg. | SIN avg. | |
|--------|---|----------|----------|--------------|
| P4 | 5 | .57 | .38 | handheld |
| P4 | 8 | .45 | .38 | tripod-mount |

The readings from the center, all from one point, are more consistent than readings from the perimeter from ten different posts. The land appeared to be filled wetland, and there might have been some magnetic materials in the fill. Table 6 shows the difference between readings from the center and from the perimeter for Suuntos and Bruntons. Again, the values shown are the averages of the standard deviations.

Table 10. Effect of reading from the center vs. reading from the perimeter of the course. All good Suunto and Brunton sets of readings.

| Location | N | LIN avg. | SIN avg. |
|-----------|----|----------|----------|
| Center | 45 | .58 | .46 |
| Perimeter | 38 | .70 | .59 |

Conclusions

If randomly chosen persons use randomly-chosen, uncalibrated, metal-bodied Suuntos to read an angle, those measurements will have a standard deviation between 0.8 and 1.4 degrees. If a Brunton-style pocket transit is used, the standard deviation will be in the range 0.3 to 1.0 degrees because the Bruntons are, or can be, better aligned with magnetic north.

The difference between what a compass considers magnetic north and actual magnetic north is a systematic error. In this analysis, actual magnetic north is approximated by Grand Average North. The standard deviation for what the Suuntos consider magnetic north was about 0.6 degrees. For the Bruntons, it was about 0.1 degrees.

In addition to the systematic magnetic north error, there are random errors caused by both the compass and the person reading the compass. The standard deviations of the random errors with the metal Suuntos range from 0.3 to 1.0 degrees. This is a factor of three difference between the best and worst readers. For Bruntons, the standard deviations of the random errors range from 0.5 to 0.9 degrees. The random errors for both Suuntos and Bruntons were reduced by reading to 0.1 degree.

Although we expected eccentricity errors in the compasses, we could not identify any. The standard deviations listed in the appendix for the linear and sinusoidal fits are generally about the same. Where the sinusoidal fit has a notably lower standard deviation, the directions calculated for the eccentricity are different. Some of the fitted curves and the residual errors were plotted. The residual errors were about as large as the sine curve.

It must be emphasized that these conclusions apply to the nearly ideal reading conditions of the study. Readings under cave conditions would have more error.

What we would do differently

Any future study should restrict itself to cave-surveying compasses. This study shows that the metal Suuntos and well-maintained Brunton pocket transits give the best performance. The Sisteco/Silva/Brunton compass might be included.

It was difficult to separate instrument effects from reader effects. In obtaining the direction a compass considered to be magnetic north, we implicitly assumed that the person doing the set of readings did not affect the average north. However, five of the better compasses were read by only one or two people. If their average appeared high, it might be because they were read by someone who consistently read high. Perhaps a better statistician could extract more information about the readers from the data in the appendix. A better-designed study should have everyone use all the instruments the same number of times. I would suggest at least two full rounds (2 sets of readings from the center and 2 sets of readings from the perimeter.) This might require using fewer instruments.

The trilateration was done as an afterthought. It should have been done each day before the readings. Because of traffic, the posts could have been brushed by a vehicle. There was an attempt to remeasure the distances more accurately with a steel tape 3 months after the study and it appeared that the distances had changed.

I tried to check for short-term variations in magnetic declination. I got the archival data from the USGS magnetic observatory at Fredericksburg Virginia, about 45 miles to the south. Unfortunately, there are no data from the time of the study. Furthermore, we neglected to record the time and date of each set of readings.

It would be worthwhile to set up a similar course where the shots are at steeper angles. The center station would have to be in a pit or on a non-magnetic tower.

Appendix 1: Results from Curve Fits

This presents the results of the linear and sinusoidal curve fits to each of the sets of ten readings. The column headings are brief to reduce their width. A more complete explanation is as follows:

- An identifying sequence number for each set
- The instrument code as previously explained
- The code for the person reading the instrument
- The location of the instrument.
 - C for center, sighting toward perimeter
 - P for perimeter, sighting toward center
- The word "bad" if the set had an error.
 - Bad sets were not used in our analyses.
- The bias for a linear fit, relative to Grand Average North
- The standard deviation for a linear fit, after bias error was removed.
- The bias for a sine curve fit.
- The standard deviation of the errors after a sine curve fit.
- The amount of eccentricity in degrees.
- The direction of the eccentricity error.
- A comment about the instrument or an error.

| Set | Inst | Person | Loc | | Linear fit | | Sine fit | | Eccentricity | | Comment |
|-----|------|--------|-----|-----|------------|------|----------|------|--------------|------|--------------|
| | | | | | Bias | Sdev | bias | Sdev | Ecc. | dir. | |
| 1 | S1 | P1 | C | | .33 | .54 | .34 | .34 | .54 | 261. | Metal Suunto |
| 2 | S1 | P1 | P | | .38 | 1.12 | .39 | .90 | .87 | 73. | |
| 3 | S1 | P2 | C | bad | 1.38 | 2.56 | 1.46 | 1.66 | 2.52 | 273. | Blunder |
| 4 | S1 | P2 | P | bad | .88 | 1.33 | .85 | 1.25 | .64 | 328. | Blunder |
| 5 | S1 | P3 | C | | .08 | .67 | .09 | .29 | .78 | 247. | |
| 6 | S1 | P3 | P | bad | .46 | .93 | .47 | .84 | .49 | 73. | Trailer |
| 7 | S2 | P3 | C | | .21 | .61 | .20 | .43 | .58 | 215. | |
| 8 | S2 | P3 | P | bad | .16 | .58 | .14 | .53 | .32 | 344. | Trailer |
| 9 | S2 | P4 | C | | -.20 | .12 | -.20 | .11 | .05 | 145. | |
| 10 | S2 | P4 | P | | .55 | .42 | .55 | .35 | .30 | 39. | |
| 11 | S3 | P5 | C | bad | 1.23 | 1.44 | 1.22 | 1.29 | .83 | 219. | Blunders |
| 12 | S3 | P5 | P | | .60 | .48 | .60 | .38 | .38 | 41. | |
| 13 | S3 | P6 | C | | .43 | .52 | .44 | .47 | .29 | 252. | |
| 14 | S3 | P6 | P | | .58 | .47 | .59 | .41 | .29 | 65. | |
| 15 | S4 | P2 | C | | -.03 | .90 | -.04 | .89 | .23 | 191. | |
| 16 | S4 | P2 | P | | .53 | .60 | .54 | .52 | .39 | 66. | |
| 17 | S4 | P3 | C | | .63 | .78 | .65 | .77 | .22 | 319. | |
| 18 | S4 | P4 | C | | -.22 | .12 | -.22 | .08 | .12 | 180. | |
| 19 | S4 | P4 | P | | .57 | .36 | .56 | .32 | .23 | 0. | |
| 20 | S4 | P4 | C | | -.13 | .17 | -.13 | .14 | .12 | 201. | |
| 21 | S4 | P4 | P | | .58 | .40 | .57 | .36 | .24 | 8. | |
| 22 | S5 | P1 | P | bad | -.52 | 1.04 | -.50 | .97 | .47 | 101. | Trailer |
| 23 | S5 | P3 | C | | -.67 | .59 | -.68 | .48 | .46 | 209. | |
| 24 | S5 | P3 | P | | -.67 | .40 | -.65 | .24 | .43 | 114. | |
| 25 | S5 | P3 | P | bad | -.47 | 1.03 | -.49 | .80 | .89 | 24. | Trailer |
| 26 | S5 | P7 | C | | -1.27 | .83 | -1.32 | .41 | 1.00 | 136. | |
| 27 | S5 | P7 | P | | -.87 | .54 | -.85 | .35 | .53 | 88. | |
| 28 | S5 | P7 | C | | -.97 | .87 | -1.02 | .53 | .96 | 139. | |
| 29 | S5 | P7 | P | | -1.82 | .97 | -1.84 | .92 | .44 | 300. | |
| 30 | S5 | P7 | C | | -.27 | .80 | -.24 | .70 | .53 | 317. | |
| 31 | S5 | P8 | C | | -2.87 | .43 | -2.88 | .37 | .31 | 143. | |

| Set | Inst | Person | Loc | Linear fit | | Sine fit | | Eccentricity | | Comment | |
|-----|------|--------|-----|------------|-------|----------|-------|--------------|------|---------|----------|
| | | | | Bias | Sdev | bias | Sdev | Ecc. | dir. | | |
| 32 | S5 | P8 | P | -2.32 | 1.17 | -2.32 | 1.04 | .72 | 48. | | |
| 33 | S5 | P9 | C | -1.02 | .60 | -1.05 | .28 | .71 | 123. | | |
| 34 | S5 | P9 | P | -.42 | .94 | -.40 | .90 | .37 | 166. | | |
| 35 | S6 | P2 | C | .31 | 1.20 | .32 | 1.14 | .46 | 55. | | |
| 36 | S6 | P2 | P | .52 | 1.12 | .51 | .98 | .72 | 37. | | |
| 37 | S6 | P3 | C | .88 | .57 | .89 | .49 | .38 | 262. | | |
| 38 | S6 | P3 | P | 1.08 | .52 | 1.08 | .41 | .43 | 44. | | |
| 39 | S7 | P2 | C | bad | -.20 | .94 | -.22 | .91 | .36 | 164. | blunder |
| 40 | S7 | P2 | P | | -.04 | .71 | -.04 | .58 | .53 | 57. | |
| 41 | S7 | P8 | C | | -1.81 | .71 | -1.79 | .56 | .57 | 273. | |
| 42 | S7 | P8 | P | | -1.68 | 1.19 | -1.66 | 1.18 | .24 | 153. | |
| 43 | S8 | P2 | C | | .39 | .96 | .41 | .85 | .60 | 21. | |
| 44 | S8 | P2 | P | | .66 | .88 | .65 | .82 | .43 | 35. | |
| 45 | S8 | P3 | C | | .88 | .57 | .89 | .48 | .40 | 249. | |
| 46 | S8 | P3 | P | bad | 1.11 | .57 | 1.10 | .51 | .35 | 27. | Trailer |
| 47 | S9 | P2 | C | bad | -.16 | 1.58 | -.19 | 1.53 | .59 | 166. | Blunder |
| 48 | S9 | P2 | P | | -.18 | .56 | -.20 | .41 | .52 | 326. | |
| 49 | S9 | P3 | C | | -.02 | .78 | .01 | .66 | .57 | 303. | |
| 50 | S9 | P3 | P | bad | .03 | .58 | .01 | .51 | .36 | 326. | Trailer |
| 51 | S9 | P4 | C | | -.13 | .46 | -.10 | .25 | .52 | 290. | |
| 52 | S9 | P4 | P | | -.14 | .32 | -.15 | .28 | .22 | 1. | |
| 53 | S9 | P7 | C | | -.23 | .55 | -.21 | .41 | .47 | 283. | |
| 54 | S9 | P7 | P | | -.80 | 1.20 | -.83 | 1.10 | .68 | 340. | |
| 55 | S9 | P8 | C | | -1.80 | .70 | -1.77 | .60 | .49 | 304. | |
| 56 | S9 | P8 | P | | -1.70 | .83 | -1.68 | .47 | .90 | 219. | |
| 57 | S10 | P2 | C | | .31 | 1.36 | .27 | 1.18 | .90 | 119. | |
| 58 | S10 | P2 | P | | -.44 | 1.22 | -.45 | 1.19 | .38 | 16. | |
| 59 | S10 | P3 | C | | .15 | .64 | .16 | .62 | .21 | 316. | |
| 60 | S10 | P3 | P | bad | .28 | .48 | .28 | .48 | .03 | 284. | Trailer |
| 61 | S10 | P4 | C | | .12 | .20 | .13 | .12 | .20 | 266. | |
| 62 | S10 | P4 | P | | .01 | .40 | .00 | .33 | .30 | 328. | |
| 63 | S10 | P7 | C | | -.20 | .44 | -.19 | .33 | .38 | 268. | |
| 64 | S10 | P7 | P | | -1.01 | .90 | -1.02 | .88 | .25 | 349. | |
| 65 | S11 | P3 | C | | -.27 | .21 | -.26 | .20 | .07 | 33. | |
| 66 | S11 | P3 | P | | .56 | .72 | .54 | .57 | .61 | 20. | |
| 67 | S11 | P9 | C | | .43 | .61 | .43 | .59 | .21 | 202. | |
| 68 | S11 | P9 | P | bad | -.02 | .90 | .00 | .88 | .23 | 146. | Trailer |
| 69 | S12 | P3 | C | | -.17 | .19 | -.17 | .19 | .05 | 256. | |
| 70 | S12 | P3 | P | bad | .78 | 1.26 | .78 | 1.23 | .35 | 248. | Trailer |
| 71 | S12 | P9 | C | | .43 | .48 | .42 | .42 | .32 | 174. | |
| 72 | S12 | P9 | P | | .28 | .76 | .30 | .72 | .37 | 161. | |
| 73 | S13 | P4 | C | | .23 | .16 | .23 | .13 | .12 | 207. | In grads |
| 74 | S13 | P4 | P | | .10 | .42 | .10 | .41 | .07 | 257. | |
| 75 | B1 | P4 | C | | -.10 | .49 | -.12 | .35 | .46 | 125. | Tripod |
| 76 | B1 | P4 | P | | .21 | .57 | .21 | .50 | .37 | 59. | |
| 77 | B1 | P4 | C | | -.09 | .30 | -.09 | .22 | .26 | 91. | |
| 78 | B1 | P4 | P | | .07 | .50 | .08 | .49 | .14 | 146. | |
| 79 | B1 | P4 | C | | .08 | .42 | .06 | .29 | .42 | 138. | |
| 80 | B1 | P4 | P | | .23 | .53 | .23 | .46 | .36 | 39. | |
| 81 | B2 | P2 | C | | -.27 | .71 | -.24 | .58 | .57 | 314. | Handheld |
| 82 | B2 | P2 | P | | -.17 | .97 | -.15 | .93 | .36 | 112. | |
| 83 | B2 | P3 | C | | -.42 | .61 | -.44 | .45 | .56 | 201. | |
| 84 | B2 | P3 | P | | .38 | .50 | .38 | .48 | .18 | 272. | |

| Set | Inst | Person | Loc | | Linear fit | | Sine fit | | Eccentricity | | Comment |
|-----|------|--------|-----|-----|------------|------|----------|------|--------------|------|----------------|
| | | | | | Bias | Sdev | bias | Sdev | Ecc. | dir. | |
| 85 | B2 | P4 | C | | -.47 | .69 | -.50 | .44 | .73 | 122. | |
| 86 | B2 | P4 | P | | .08 | .77 | .09 | .43 | .83 | 67. | |
| 87 | B2 | P4 | C | | -.25 | .45 | -.26 | .39 | .32 | 173. | |
| 88 | B2 | P4 | P | | .13 | .59 | .14 | .42 | .53 | 81. | |
| 89 | B3 | P4 | C | | -.10 | .30 | -.10 | .23 | .25 | 87. | Tripod |
| 90 | B3 | P4 | P | | .07 | .50 | .08 | .49 | .14 | 146. | |
| 91 | B4 | P5 | C | | -.64 | .67 | -.61 | .34 | .78 | 25. | Handheld |
| 92 | B4 | P5 | P | | .37 | .82 | .34 | .50 | .89 | 18. | |
| 93 | B4 | P5 | C | | -.10 | 1.09 | -.10 | .81 | .94 | 61. | |
| 94 | B4 | P5 | P | | .20 | .46 | .18 | .31 | .48 | 10. | |
| 95 | B5 | P3 | C | | -.02 | .88 | -.03 | .83 | .39 | 213. | Handheld |
| 96 | B5 | P3 | P | | .48 | .60 | .45 | .44 | .57 | 323. | |
| 97 | B5 | P4 | C | | -.77 | .34 | -.78 | .23 | .34 | 203. | |
| 98 | B6 | P1 | C | bad | 2.13 | 4.97 | 2.08 | 4.57 | 2.51 | 80. | Mils Brunton |
| 99 | B6 | P1 | P | bad | 1.18 | 2.46 | 1.23 | 2.39 | .80 | 135. | |
| 100 | B6 | P3 | C | bad | 1.67 | 5.05 | 1.63 | 4.73 | 2.25 | 77. | |
| 101 | B6 | P3 | P | bad | .89 | 2.47 | .93 | 2.40 | .83 | 136. | |
| 102 | B6 | P4 | C | bad | -1.04 | 4.50 | -1.05 | 4.35 | 1.54 | 227. | |
| 103 | PL1 | P3 | C | | .43 | 1.00 | .46 | .83 | .77 | 15. | Plastic Suunto |
| 104 | PL1 | P3 | P | | .73 | 1.03 | .79 | .59 | 1.13 | 115. | |
| 105 | PL1 | P6 | C | | -.17 | .77 | -.16 | .60 | .63 | 252. | |
| 106 | PL1 | P6 | P | | .28 | .39 | .26 | .26 | .41 | 0. | |
| 107 | PL2 | P2 | C | | .23 | .89 | .20 | .72 | .74 | 183. | |
| 108 | PL2 | P2 | P | | .28 | 1.14 | .29 | .68 | 1.18 | 66. | |
| 109 | PL2 | P3 | C | | .53 | .86 | .54 | .81 | .36 | 255. | |
| 110 | PL2 | P3 | P | | .33 | .75 | .35 | .67 | .47 | 202. | |
| 111 | PL2 | P5 | C | | .34 | .41 | .33 | .33 | .33 | 203. | |
| 112 | PL2 | P5 | P | | .28 | .61 | .32 | .34 | .71 | 181. | |
| 113 | PL2 | P6 | C | | .28 | 1.09 | .29 | .47 | 1.27 | 242. | |
| 114 | PL2 | P6 | P | | .28 | .68 | .25 | .37 | .75 | 284. | |
| 115 | PL3 | P2 | C | | .13 | .94 | .09 | .71 | .85 | 135. | |
| 116 | PL3 | P2 | P | | .03 | 1.42 | .05 | 1.16 | 1.05 | 79. | |
| 117 | PL3 | P3 | C | | .38 | .99 | .38 | .98 | .14 | 124. | |
| 118 | PL3 | P3 | P | | .18 | .84 | .20 | .80 | .37 | 133. | |
| 119 | PL3 | P5 | C | | .26 | .76 | .23 | .57 | .71 | 172. | |
| 120 | PL3 | P5 | P | | .22 | .65 | .25 | .51 | .57 | 178. | |
| 121 | PL3 | P6 | C | | -.22 | .90 | -.20 | .46 | .99 | 256. | |
| 122 | PL3 | P6 | P | | -.07 | .79 | -.10 | .51 | .79 | 281. | |
| 123 | PL4 | P3 | C | | .83 | .91 | .82 | .89 | .26 | 140. | |
| 124 | PL4 | P3 | P | | 1.23 | 1.15 | 1.24 | .82 | 1.03 | 63. | |
| 125 | PL4 | P6 | C | | .68 | 1.57 | .68 | .34 | 1.99 | 237. | |
| 126 | PL4 | P6 | P | | .53 | .57 | .53 | .32 | .60 | 237. | |
| 127 | PL5 | P1 | C | | 1.18 | 1.12 | 1.21 | .87 | .95 | 26. | |
| 128 | PL5 | P1 | P | | 1.18 | .95 | 1.19 | .64 | .90 | 64. | |
| 129 | PL5 | P3 | C | | .68 | 1.13 | .66 | .87 | .94 | 83. | |
| 130 | PL5 | P3 | P | | .88 | .87 | .91 | .49 | .93 | 89. | |
| 131 | PL5 | P3 | P | | 1.23 | 1.04 | 1.23 | .55 | 1.15 | 54. | |
| 132 | PL5 | P7 | C | | -.27 | .94 | -.27 | .75 | .74 | 62. | |
| 133 | PL5 | P7 | P | | .83 | .63 | .81 | .49 | .53 | 311. | |
| 134 | PL5 | P7 | C | | .45 | .47 | .46 | .46 | .11 | 300. | |
| 135 | PL5 | P7 | P | | .47 | 1.01 | .41 | .49 | 1.23 | 358. | |
| 136 | PL5 | P8 | C | | -.90 | .79 | -.88 | .33 | .94 | 41. | |
| 137 | PL5 | P8 | P | | -.39 | 1.44 | -.42 | .93 | 1.47 | 36. | |
| 138 | PL5 | P9 | C | | .38 | .53 | .40 | .44 | .41 | 349. | |

| Set | Inst | Person | Loc | Linear fit | | Sine fit | | Eccentricity | | Comment | |
|-----|------|--------|-----|------------|------|----------|------|--------------|------|---------|-----------------|
| | | | | Bias | Sdev | bias | Sdev | Ecc. | dir. | | |
| 139 | M1 | P4 | C | | | | | | | | |
| 140 | M1 | P4 | P | | | | | | | | Hockey Puck |
| 141 | M1 | P9 | C | | | | | | | | |
| 142 | M1 | P9 | P | | | | | | | | |
| 143 | M2 | P2 | C | | | | | | | | |
| 144 | M2 | P2 | P | | | | | | | | Omega |
| 145 | M2 | P4 | C | | | | | | | | |
| 146 | M2 | P4 | P | | | | | | | | |
| 147 | M3 | P4 | C | | | | | | | | |
| 148 | M3 | P4 | P | | | | | | | | Meridian |
| 149 | M4 | P2 | C | bad | | | | | | | |
| 150 | M4 | P2 | P | bad | | | | | | | Ritchie |
| 151 | M4 | P4 | C | | | | | | | | |
| 152 | M4 | P4 | P | bad | | | | | | | |
| 153 | M5 | P1 | C | | | | | | | | |
| 154 | M5 | P1 | P | | | | | | | | Silva Type 50 |
| 155 | M6 | P1 | C | bad | | | | | | | |
| 156 | M6 | P1 | P | bad | | | | | | | Weems & Plath |
| 157 | M6 | P2 | C | bad | | | | | | | |
| 158 | M6 | P2 | P | bad | | | | | | | |
| 159 | M7 | P1 | C | bad | | | | | | | |
| 160 | M7 | P1 | P | bad | | | | | | | Wilkie lensatic |
| 161 | M7 | P2 | C | bad | | | | | | | |
| 162 | M7 | P2 | P | bad | | | | | | | |